A New Scenario of Confinement and Hadron Spectra

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This work presents a new phenomenological description of QCD vacuum where the color background field is depicted by a constant parallel vector with amplitude proportional to $\sigma \sim 0.28GeV^2$. The familiar Regge relation can be derived directly in a classic meaning. A new mass formula of hadrons are conjectured to give a very consistent description of hadron mass spectra. The lower bounds of the baryon mass are determined to be $\sqrt{3\sigma}$ for nucleon and $\sqrt{5\sigma}$ for isospin-$\frac{3}{2}$ baryon. In the meanwhile, the mass-square differences of 1^3S_1 and 1^1S_0 meson states are obtained to be 2σ. We also predict the mass of the lowest four-quark state to be $\sqrt{4\sigma}$, which favors the $a_0(980)$ and $f_0(980)$ to be four-quark candidates. On the other hand, the effective heavy quark confinement potential is a direct result of this model.

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Quantum Chromodynamics(QCD) is commonly accepted as the correct theory for strong interaction. In contrast to its success in the high energy regime, there are many questions unsolved in the low energy sector due to its nonperturbative characteristics. Specifically, confinement is still a conjecture and can not be derived from the first principle. Various QCD-based models were supposed to describe the hadron spectrum but the results are always model dependent. Apart form the confinement, there are other open questions to be answered, such as why the lightest baryon, say, nucleons, are so heavy, since they are composed of light quarks, how can many mesons and baryons be arranged on Regge trajectories $J = \alpha + \alpha' M^2$ (J and M are the angular momentum and the mass of the hadron, respectively, and the parameters α and α' are the interception and the Regge slope) [1, 2], how to explain the approximate constant mass-square difference [3] of 1^3S_1 and 1^1S_0 meson states? The aim of this work is to give a tentative interpretations of these puzzles with a new model.

It is a natural assumption that the whole world should be a color singlet, since there are no isolated color charges found from experiments. Therefore, theoretically any isolated color charge should be considered with an implicit condition that there must exist other color charges in the same time so that all the charges add up to a singlet. On the other hand, QCD vacuum is topologically non-trivial and has non-zero gluon condensate and quark condensate, which is in contrast with the case of QED. As a result of above, when the behavior of a color charge is considered, the QCD vacuum can be viewed as a background color field (called *colored vacuum* here) interacting with the charge considered, due to the existence of other color charges. Or in other words, the universal property of this 'color coherence' implies that color charges should be considered in '(generalized) hadron systems', but there are no limits assumed on the sizes of these 'hadrons' at present. As a model, we choose the hadron rest frame as the preferred reference frame to study the motions of color charges in this work, and require that color charges obey relativistic kinetics.

Intuitively and in the classical meaning, vacuum should be spatially homogeneous, or in other words, the color electric field $\mathbf{E}$ and the color magnetic field $\mathbf{B}$ felt by color charges in the vacuum should be identical, if the inter-charge Coulomb type interaction is ignored at large enough separation. This kind of vacuum can be described by an accessory quantity $\hat{\hat{\sigma}}$, called vacuum tensor, which is a constant spatial parallel vector and is invariant in spatial translation and rotation,

$$\hat{\hat{\sigma}} = \sigma\delta_{ij}\hat{\mathbf{x}}_i\hat{\mathbf{x}}_j, \tag{1}$$

where $\hat{\mathbf{x}}_i$ is the unit vector of the spatial direction i. Its most attractive feature is that it has uniform projection in any direction. For example, its projection in the direction of $\hat{\mathbf{r}} = (sin\theta cos\phi, sin\theta sin\phi, cos\theta)$ is

$$\sigma(\theta,\phi) \equiv \hat{\hat{\sigma}} \cdot \hat{\mathbf{r}} = \sigma\hat{\mathbf{r}}. \tag{2}$$

In a definite frame of reference and a definite reference point, there are two definite directions for a color charge moving in the vacuum, say, the directions of the velocity $\hat{\mathbf{v}}$ and the angular momentum of the charge $\hat{\mathbf{L}}$. Purely artificially, we define the color electric field strength felt by the color charge in the vacuum as $\mathbf{E} = \hat{\hat{\sigma}} \cdot \hat{\mathbf{v}}$ and the magnetic field strength as $\mathbf{B} = \hat{\hat{\sigma}} \cdot \hat{\mathbf{L}}$. In analogy with electromagnetic theory, the equation of motion of the color charge in the colored vacuum is written as

$$\frac{d\mathbf{p}}{dt} = -\sigma\frac{\mathbf{v}}{|\mathbf{v}|} - \sigma\mathbf{v} \times \hat{\mathbf{L}}, \tag{3}$$

where $\mathbf{v}$ is the velocity and the minus signs are chosen to guarantee the conservations of energy and angular momentum (see below). It is obvious from the equation that angular momentum and the energy of the charge vary

v_0	$(1-v_0^2)^{-\frac{1}{2}}$	$\Delta S/R$	$R/\sqrt{L_2}$
0.5	1.155	1.008	1.426-1.47
0.9	2.294	1.059	1.377
0.99	7.089	1.202	1.420
0.999	22.37	1.310	1.415
0.9999	70.71	1.390	1.410
0.99999	223.6	1.411	1.422
0.999999	707.1	1.414	1.415
~ 1	$\sim \infty$	$\sim \sqrt{2}$	$\sim \sqrt{2}$

TABLE I: The numerical solution of Eqn.3. The initial energy of a massive color charge is proportional to $(1-v_0^2)^{-\frac{1}{2}}$.

during the moving in the vacuum. For a single quark moving away from a place very close to a specific point O, the variation of angular momentum referring to O at time τ is

$$\begin{aligned}\Delta L &= \int_0^\tau \mathbf{r} \times \frac{d\mathbf{p}(\mathbf{t})}{dt} dt \qquad (4)\\ &= -\frac{1}{2}\sigma r(\tau)^2 - \sigma \int_0^\tau dt \mathbf{r}(t) \times \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|}\\ &\equiv -\frac{1}{2}\sigma R^2 - \sigma L_2,\end{aligned}$$

and the energy variation is

$$\Delta E = -\sigma \int_0^\tau v dt \equiv -\sigma \Delta S. \qquad (5)$$

Given the initial velocity v_0 and the final velocity $v \sim 0$ (in the practical calculation, the final velocity is set to be $v_f = 0.001$), the Eqn. 3 can be solved numerically with the results shown in the table.

Table implies a very interesting relation for an ultra-relativistic color charge moving in the QCD vacuum

$$\frac{\Delta S}{R} = \sqrt{2} \quad for \quad v_0 \to 1. \qquad (6)$$

Now we apply the above logic to a $q\bar{q}$ meson state (in the paper q denotes the a light quark, while Q denotes a heavy quark). In the meson's rest frame, imagining the constituent quark-antiquark pair is excited with initial energy and momentum $(E, \pm\mathbf{p})$ and the initial angular momentum $\mathbf{J}$ referring to the center of mass. According to the equation of motion, they will exchange energy with the vacuum during their flight and will finally be quasi-static, thus the meson is made up of the quasi-static quark pair plus the excited vacuum (after absorbing the energy and angular momentum from the quarks). With the fact that the mass of light quark is $m_q \sim 5MeV$ and the mass of a typical light meson is $M \sim 1-2GeV$, the initial velocity of a light quark is $v_0 > 0.99995$ and the relation Eq.6 is approximately correct. If we assume that the initial position of the quark pair is very near the center of mass and the final separation is $2R$, the mass M of the meson is

$$M = 2E = 2m_q + 2\sigma\Delta S \approx 2\sqrt{2}\sigma R, \qquad (7)$$

while the total angular momentum J is

$$J = 2|\Delta L| = \sigma R^2 + 2\sigma(\frac{R}{\sqrt{2}})^2 = 2\sigma R^2. \qquad (8)$$

M and J satisfy the relation

$$J = \frac{1}{4\sigma} M^2. \qquad (9)$$

It is encouraging that we can obtain the Regge relation even in so simple a classic model. However, the equation of motion implies that the constituents be static finally which is of course not the fact in the real world. To give a more delicate description of hadron in quantum theory, we put forward the following assumptions:

i) Hadrons are strongly coupled system made up of constituents (such as quarks and gluons) and the excited color vacuum.

ii) Constituents reside in stationary states in hadrons.

iii) As far as a constituent is concerned, other constituent(s) and the vacuum act effectively as a background field which can be described by the parallel vector defined above.

iv) Since constituents are in stationary states, the electric-type interactions in Eqn.3 can be omitted. Thus constituents in hadrons are analogous to electrons moving in a identical magnetism.

We assume the stationary equation of a constituent in a hadron is

$$E^2\phi = (m^2 + (\mathbf{p} + \mathbf{A})^2)\phi, \qquad (10)$$

where ϕ is the state function, m is the mass of the constituent, and E is the quasi-energy (see below). $\mathbf{A}$ is the 'vector potential' due to the color magnetic field of the vacuum. If we choose the direction of the angular momentum of a hadron (also the direction of the color magnetic field) as z-axis, the center of mass as the reference point, and write the vector potential as $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$, we have,

$$(E^2 - m^2)\phi = [\mathbf{p}^2 + \frac{1}{4}\sigma^2(x^2 + y^2) - \sigma L_z]\phi, \qquad (11)$$

where $L_z = xp_y - yp_x$. The eigenvalues can be directly read out as

$$E_n(p_z)^2 = m^2 + p_z^2 + \sigma(2n + 1). \qquad (12)$$

We call E_n the quasi-energy because it is meaningless to say the energy of a constituent in a strongly coupled system, say, a hadron. Instead we introduce a phenomenological mass formula for a multi-quark system (

in the hadron rest frame)

$$M^2 = \sum_i (E_n^i)^2 = \sum_i m_i^2 + \sigma \sum_i (2n_i + 1), \tag{13}$$

which can describe many properties of hadron spectra as follows.

Mesons For a meson composed of two light constituent quarks in its rest frame, the average momenta of the two constituents should be equal, so that n_1 and n_2 in Eqn.13 take the same value n and the mass formula becomes

$$M^2 = \Delta^2(m_1, m_2) + 2(2n+1)\sigma, \tag{14}$$

where $\Delta^2(m_1, m_2)$ is the mass term comes from other mechanisms, such as chiral symmetry breaking. We argue that $\Delta(m_1, m_2)$ is equal to the mass of pseudoscalar counterpart, M_{0^-}, and rewrite the above formula as

$$M^2 = M_{0^-}^2 + 2(2n+1)\sigma. \tag{15}$$

The second term is obviously the vacuum effect and comes from the vacuum excitations. To comply with the classic scenario discussed above, we argue additionally that n is the angular momentum carried by the vacuum excitations and is denoted with J_v from now on. Maybe we can go further to assume that the quantum of the vacuum excitation is with $J^{PC} = 1^{++}$ and contributes 4σ to the M^2. The main results of this mass formula is:

i) Pseudoscalars, such as π, K, etc, are Goldstone particles due to the chiral symmetry breaking, and are excluded in the scenario of this work.

ii) The mass-square differences of the lowest $q\bar{q}$-mesons ($J_v = 0$) in this model, say, ρ, K^*, etc, and their psuedoscalar counterparts are $M^2 - M_{0^-}^2 = 2\sigma$, which is flavor independent. This gives a natural explanation of the phenomenon that experimentally the mass-square differences of $^3S_1$ states and the corresponding $^1S_0$ states are approximately constant, namely, $0.55GeV^2$, for $q\bar{q}$ and $q\bar{Q}$ mesons. This is not applied to $Q\bar{Q}$ system. A possible reason is that the inter-quark Coulomb interaction plays an important role in $Q\bar{Q}$ system, since the Coulomb effects is proportional to the reduced mass $\mu = m_Q/2$. While in the case of $q\bar{q}$ and $q\bar{Q}$, the reduced mass is $\mu \sim m_q \sim 0$.

iii) Masses of mesons made up of light constituents are dominated by the vacuum effects, especially for large J_v. The experimental implication of this fact is that light mesons (or more generally, hadrons) can be sorted into various 'mass bands'. Mesons in the same band have similar masses which are around the mass value $\sqrt{(M_{0^-}^2 + 2\sigma) + 4\sigma J_v}$, and the variations from this value are results of other mechanisms, such as Coulomb interaction, orbit-spin coupling, etc. In other words, we can take this mass formula as a generalized Regge relation,

$$J_v = \alpha + \alpha' M^2, \tag{16}$$

with the Regge slope $\alpha' = 1/(4\sigma)$ and the interception $\alpha = -M_{0^-}^2 - 2\sigma$. Using the experimental value $\alpha'^{-1} \sim 1.1GeV^2$ we obtain the value of $\sigma \approx 0.28GeV^2$, which can be also determined from the mass-square differences between 1^3S_1 and 1^1S_0 meson states, $\Delta M^2 \sim 0.55GeV^2 = 2\sigma$.

For simplicity, we choose strange mesons for example to illustrate the scenario described above, since there exists many non-$q\bar{q}$ candidates in the unflavored meson spectra.

We maintain the P parity and the C parity assignments $P = (-)^{L-1}$ and $C = (-)^{L+S}$ for mesons. $J_v = 0$ state corresponds to a mass $M = 0.891GeV$, complies with $K^*(1^-)(892)$. $J_v = 1$ corresponds to a mass $M = 1.387GeV$. There are four strange mesons around this mass value, such as $K_1(1^+)(1400)$, $K_0^*(0^+)(1430)$, $K^*(1^-)(1410)$, and $K_2^*(2^+)(1430)$. K_2^*, K_1, K_0^* may be sorted into a ($J_v = 1, L = 1$) triplet, K^* might be a state of ($J_v = 1, L = 0$). $J_v = 2$ corresponds to a mass $M = 1.744GeV$. There are five strange mesons in this range, which are $K^*(1^-)(1680)$, $K_2(2^-)(1770)$, $K_3^*(3^-)(1780)$, $K_2(2^-)(1820)$, and $K(0^-)(1830)$. These states might be sorted into a multiplet ($J_v = 2, L = 0$, or 2). $J_v = 3$ corresponds to a mass $M = 2.040GeV$. There are three strange mesons in this range, which are $K_4^*(4^+)(2.045)$, $K_s^*(2^+)(1980)$ and $K_0^*(0^+)(1950)$. The first two mesons might be the candidates for ($J_v = 3, L = 1$) triplet, while the third might be ($J_v = 3, L = 3$). The strange meson $K(3^+)(2320)$ might be a state of ($J_v = 4, L = 1$).

Baryons Similarly, the mass formula of baryons can be written as

$$M_b = \sqrt{\delta(m) + (2n_1 + 2n_2 + 2n_3 + 3)\sigma}. \tag{17}$$

If the constituents are all light quarks, δ is small and can be ignored temporarily, the lowest mass of a isospin-$\frac{1}{2}$ baryon is approximately $\sqrt{3\sigma} = 0.918GeV$, which implies that nucleons are candidate states of ($n_1 = 0, n_2 = 0, n_3 = 0$). The first excited state has the mass $M = \sqrt{7\sigma} = 1.4GeV$ and favors the Roper to be its candidate. For the isospin-$\frac{3}{2}$ $\Delta^{++}(1232)$, the three constituents reside in the same isospin state $|II_3\rangle = |\frac{1}{2}\frac{1}{2}\rangle$. Taking into account that the spin of each constituent have two possible values $s_z = \frac{1}{2}, -\frac{1}{2}$, Pauli's exclusion principle requires that n_1, n_2, n_3 can not take the same value simultaneously. Thus the lowest state for isospin-3/2 baryon should be $(n_1, n_2, n_3) = (1,0,0), (0,1,0)$, or $(0,0,1)$ with the mass $M_b = \sqrt{5\sigma} = 1.18GeV$.

It is remarkable that the color degree of freedom was introduced originally to solve the spin-statistics 'puzzle' of Δ^{++} [5] and hadrons are thus conjectured to be color singlets. However in our logic, this is not a conjecture but a consequence of the following deduction: since the universe is a color singlet, a non-singlet system must be accompanied by other non-singlet system(s), and all the systems make up a bound state with energy described by Eqn. 13, which increases with the separations between

these systems (see below). By the way, the color index of a quark always changes after interacting with gauge field and may not be taken as a good quantum number of a quantum state. As far as this is concerned, our explanation of the Δ^{++} spin-statistics 'puzzle' seems more reasonable. This interpretation also implies that the spin of Δ might not be contributed totally by quark spins.

From the discussion above, we can get two important mass ratios $M_N/M_\rho \approx \sqrt{3/2} \approx 1.22$ and $M_\Delta/M_N = \sqrt{5/3} = 1.29$ (where M_N,M_ρ, and M_Δ are the masses of nucleon,ρ meson, and Δ baryon, respectively), which are in very good agreement with the experiment values 1.22 and 1.31.

If our logic above is correct, the lowest mass of four quark state $qq\bar{q}\bar{q}$ can be predicted to be $M \approx \sqrt{4\sigma} = 1.060GeV$. Experimentally, scalar mesons $a_0(980)$ and $f_0(980)$ lie in this range and have been suggested to be candidates of four-quark state.

Confinement Confinement is also a directly result of our model. Taking $q\bar{q}$ system for instance, if a constituent resides in the stationary state $|n\rangle$, its average distance from the mass center, or 'cyclotron radius', can be derived semi-classically as [4]

$$r = \sqrt{\langle x^2 + y^2 \rangle} = \sqrt{\frac{(2n+1)}{\sigma}}. \tag{18}$$

The relation between the energy of the system and the average separation d of the two light quarks is

$$M = \sqrt{M_{0^-}^2 + 2(2n+1)\sigma} \approx \sigma\sqrt{2}\sigma r \equiv \frac{\sqrt{2}}{2}\sigma d, \tag{19}$$

which means the energy of the system is proportional to the distance of two constituents, as is the exact meaning of confinement.

For heavy quarkonium, we assume that the quarks are so heavy that they are almost decoupled from the vacuum gauge field and write the energy of the system as

$$\begin{aligned} E &= 2M_Q + \sqrt{2(2n+1)\sigma} = 2M_Q + \sqrt{2}\sigma r \\ &\equiv 2M_Q + \sigma_{Q\bar{Q}} d, \end{aligned} \tag{20}$$

where $\sigma_{Q\bar{Q}} = \sqrt{2}/2\sigma \approx 0.2GeV^2$, the so-called string tension, is in agreement with the commonly used value. The three quark potential can be similarly derived,

$$E = 3M_Q + V_{3Q} = 3M_Q + \sigma\sqrt{r_1^2 + r_2^2 + r_3^2}. \tag{21}$$

If we denote $L_{min} = (r_1 + r_2 + r_3)$ and write V_{3Q} as $\sigma_{3Q} L_{min}$ (in flux tube model, L_{min} is the minimal value of the total length of Y-type color flux tubes linking three quarks), we have,

$$\sqrt{\frac{2}{3}} \approx 0.82 \leq \frac{\sigma_{3Q}}{\sigma_{Q\bar{Q}}} \leq 1, \tag{22}$$

which is exactly the result of lattice QCD where $\sigma_{3Q} \sim 0.9\sigma_{Q\bar{Q}}$ [6] (the authors of [6] argue that $\sigma_{3Q} \approx \sigma_{Q\bar{Q}}$ be a universal relation).

In summary, starting from a simple classical scenario, we developed a model for hadron systems. A hadron can be viewed as a strongly coupled system of constituents and the vacuum background field which can be described by a parallel vector. In this model, the Regge relation and the confinement can be neatly derived, and the hadron mass spectra can be consistently described by a simple mass formula. The most interesting result of this model is that the the Regge slope, the string tension parameter in the effective heavy-quark potential, and the mass-square differences between 1^3S_1 and 1^1S_0 meson states, can be represented by a single constant, $\sigma \approx 0.28GeV^2$, which is the amplitude of the colored vacuum background field.

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